Homework set 1, Phys 785

Reading: 11.1-4, 11.6-7

All chapters (Ch.), equations (Eq.) and problems (Pb.) refer to Jackson 3rd edition.

Problem 1

Velocity and acceleration.

a) A reference frame K' is moving in the x direction with respect to reference frame K. Write down the Lorentz transformation for this case, and use it to find the transformation rule for velocity. Explain your notation and show intermediate steps. (The general result is given in Eq. 11.31.)

b) Since the usual velocity or acceleration does not transform as a Lorentz vector, we construct Lorentz vector $U^\alpha = dx^\alpha/d\tau$ and $W^\alpha = dU^\alpha/d\tau$, where τ is the proper time. Compute the inner product $U^\alpha U_\alpha$ and $U^\alpha W_\alpha$, and show that both yield constants. (Hint: you can do this in a convenient reference frame.)

[5+5 points]

Problem 2

Relativistic Doppler shift. The frequency and wave number of a plane E&M wave together form a Lorentz vector (see Eq. 11.29, 11.20, and read the text if needed). Consider E&M plane wave of wavelength 700nm traveling north from the Vienna metro station. To a passenger in a high speed train going east with half the speed of light ($c/2$), what direction does the light travel, and what is its wave length? Is it red-shifted or blue shifted? [10 points]

Problem 3

Field strength tensor. The electric and magnetic fields are part of the contravariant field strength tensor, $F^{\alpha\beta}$, defined by, in matrix from,

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

For example, $F^{01} = -E_x = -F^{10}$. Using the rules of tensor analysis, find the covariant tensor $F_{\alpha\beta}$, and the dual tensor $\mathcal{F}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta}$, and show the final result in matrix form. Compute the contraction $F_{\alpha\beta}F^{\alpha\beta}$ and $F^{\alpha\beta}\mathcal{F}_{\alpha\beta}$. [10 points]