Homework set 3, Phys 785, Spring 2014

Reading: Ch. 12.11, 9.1-4

All chapters (Ch.), equations (Eq.) and problems (Pb.) refer to Jackson 3rd edition.

Problem 1

Radiation fields of rotating charge. A single charge q is rotating about the origin in the $x-y$ plane in a circle of radius R at constant angular velocity ω.

(a) Compute the electric dipole $\mathscr{P}(t)$ of the system, and show it is proportional to $\mathrm{Re}[(\hat{x}+i\hat{y})e^{-i\omega t}]$ up to some constant phase factor. Notice that if we write $\mathscr{P}(t) = \mathbf{p}e^{-i\omega t}$, then $\mathbf{p}$ is complex in this case.

(b) The vector potential $\mathbf{A}$ in the far zone is given by Eq. (9.16), with $\mathbf{p}$ given by result of (a). From $\mathbf{A}$, compute the *radiation fields* $\mathbf{H}$ and $\mathbf{E}$ (i.e., only keep terms to order $1/r$).

(c) Use Eq. (9.21) and results of (b) to compute the average radiation power per unit solid angle $dP/d\Omega$. Compare your result with Eq. (9.23) where $\mathbf{p}$ is assumed to point at some fixed direction. [20 points]

Problem 2

Multipole moments of linear antenna. A thin linear antenna of length d is excited in such a way that the sinusoidal current makes a full wavelength of oscillation as shown in the figure of Jackson Problem 9.16, i.e.,

$$\mathbf{J} = \hat{z} I_0 \sin(\frac{2\pi}{d} z)\delta(x)\delta(y)e^{-i\omega t}, \quad -d/2 \le z \le d/2$$

(a) Calculate the electric dipole, magnetic dipole, and electric quadrupole of the system.

(b) Neglect higher order (beyond magnetic dipole and electric quadrupole) contributions, compute the total radiation power P, and show that $P = RI_0^2/2$. Determine the *radiation resistance* R (in unit of ohm). You may use for example Eq. (9.24), (9.49) directly.

[20 points]

Problem 3 is extra credit. You can choose to play with it if you got time.

Problem 3

Rotating pulsars. Pulsars are highly magnetized, rotating neutron stars that emit electromagnetic radiation. Consider a pulsar with magnetic moment $\mathbf{m}$, which makes an angle θ with the rotation axis $\hat{z}$. The rotation is slowing down due to radiation from the rotating $\mathbf{m}$, so the angular velocity $\omega(t)$ decreases over time t. Suppose the decrease in ω in one turn of rotation is small.

(a) Only the component of $\mathbf{m}$ perpendicular to the rotation axis, $\mathbf{m}_\perp$, is oscillating and contributes to radiation. Find $\mathbf{m}_\perp$ (it is a rotating vector on the xy plane) and compute the total radiation power P.

(b) Energy conservation dictates that $P = -d(I\omega^2/2)/dt$, where $I\omega^2/2$ is the rotational kinetic energy and I is the moment of inertia about the axis of rotation. Use this to find the equation of motion for $\omega(t)$.

(c) Solve the equation of motion for $\omega(t)$. Assume $\omega(t=0) = \omega_0$.

(20 points)